Quantum Key-Revocable Dual-Regev Encryption, Revisited

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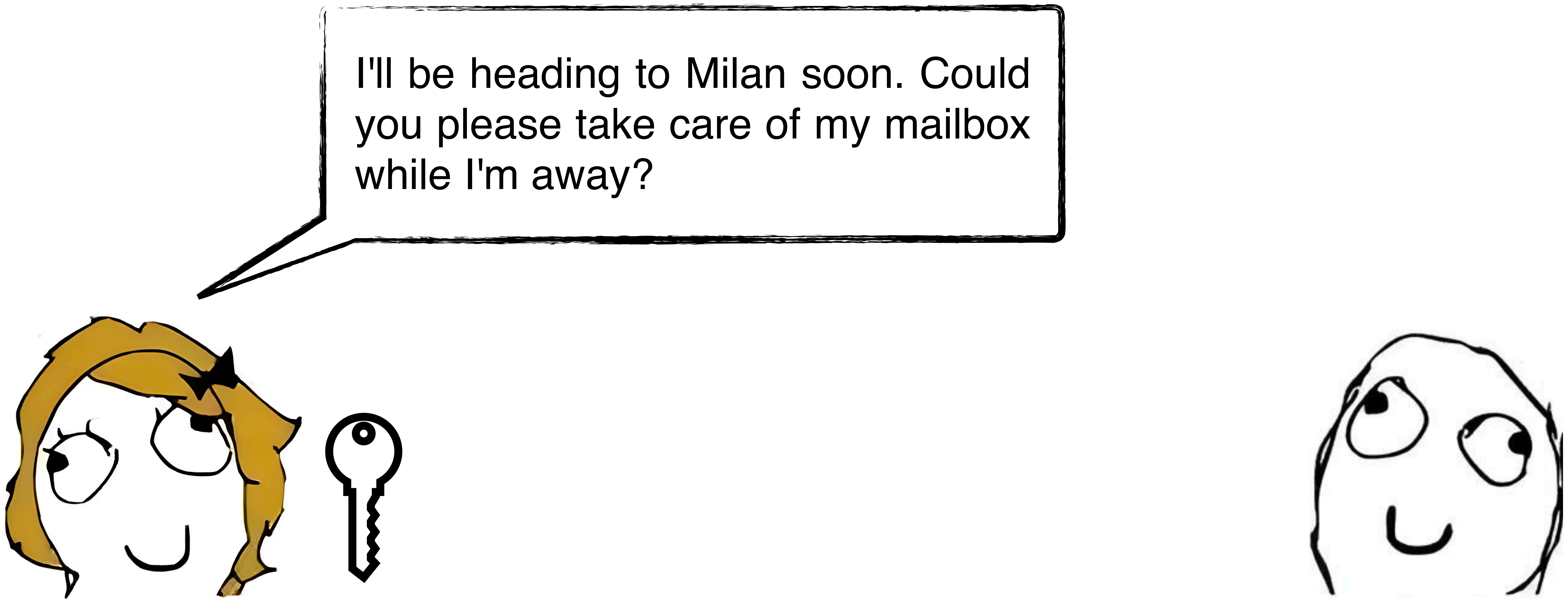
Sometimes we want to delegate and then revoke…



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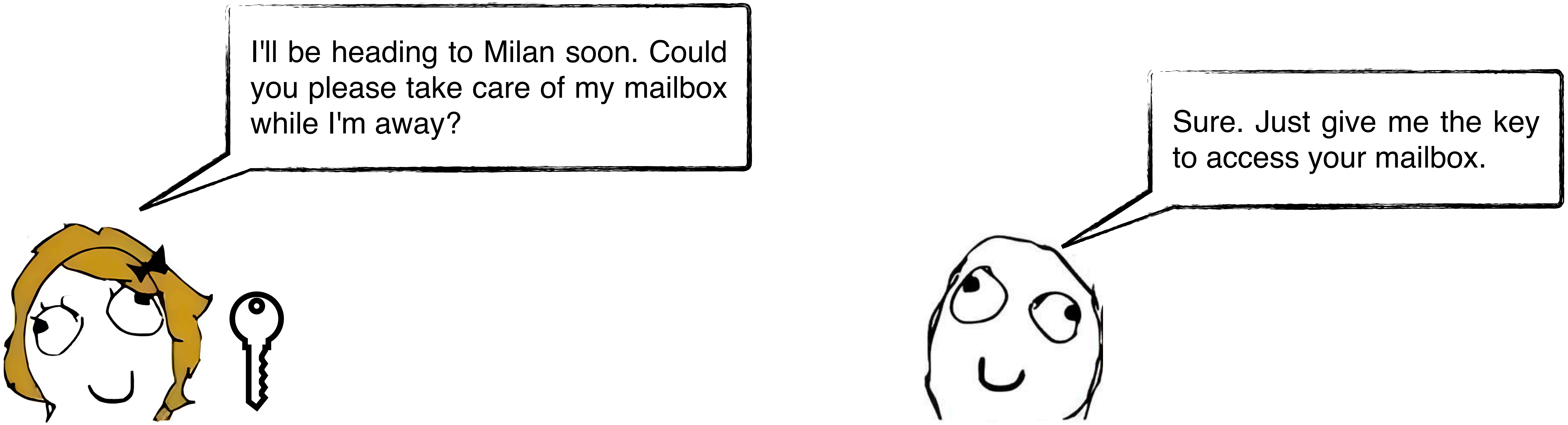

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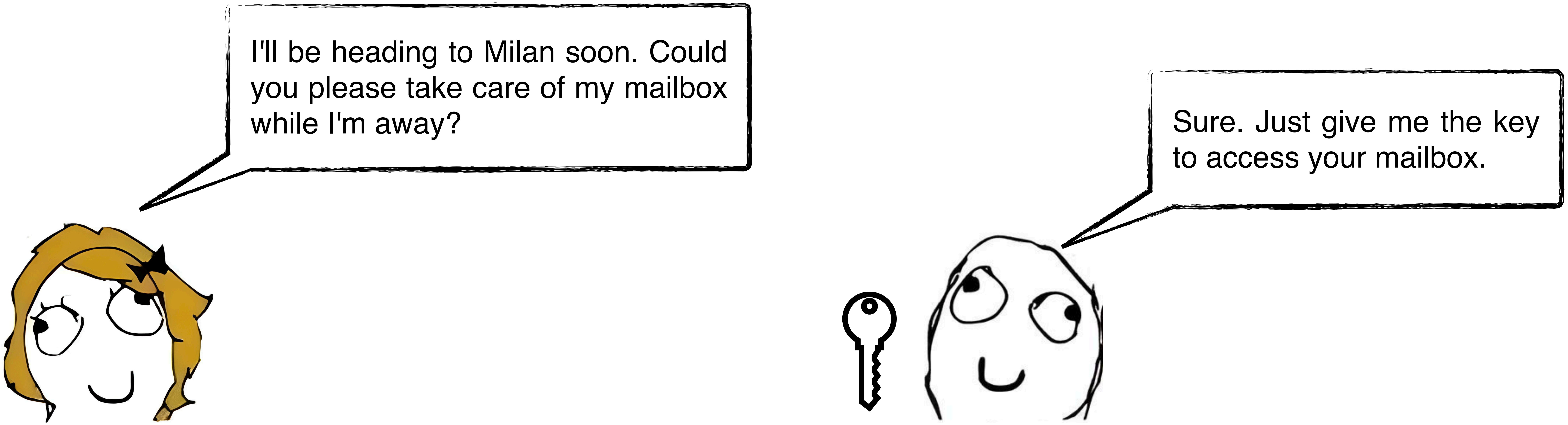

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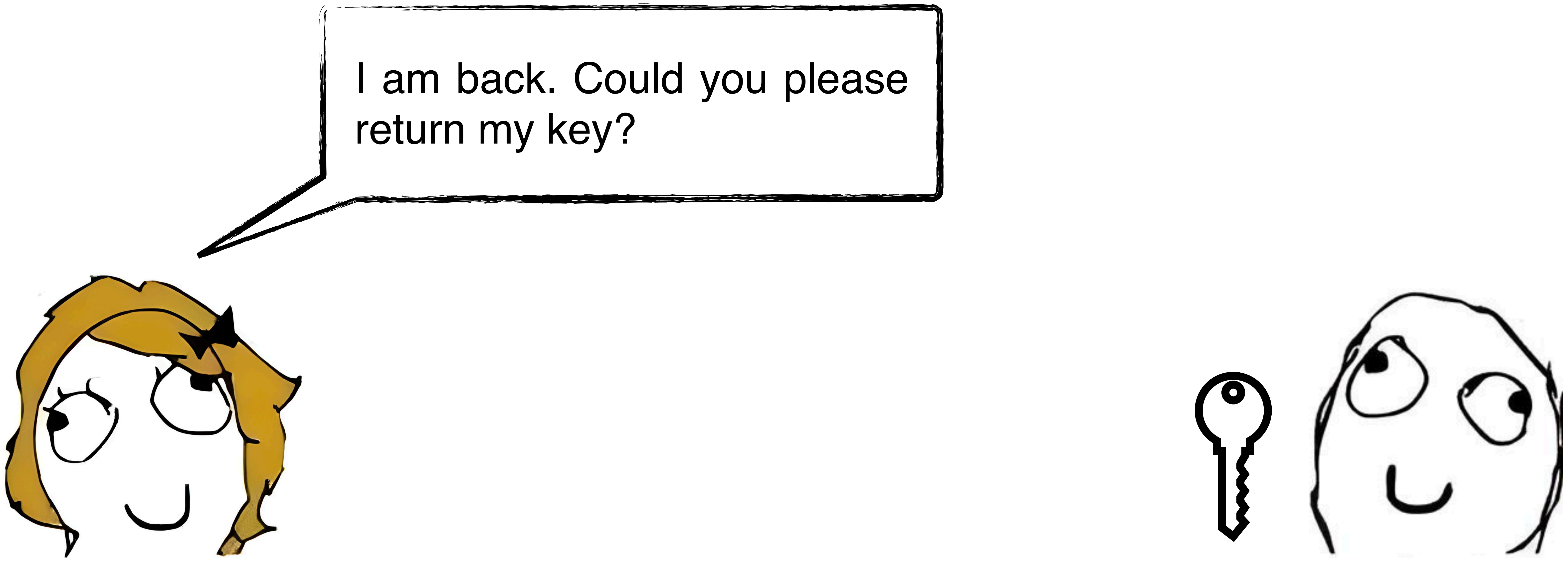

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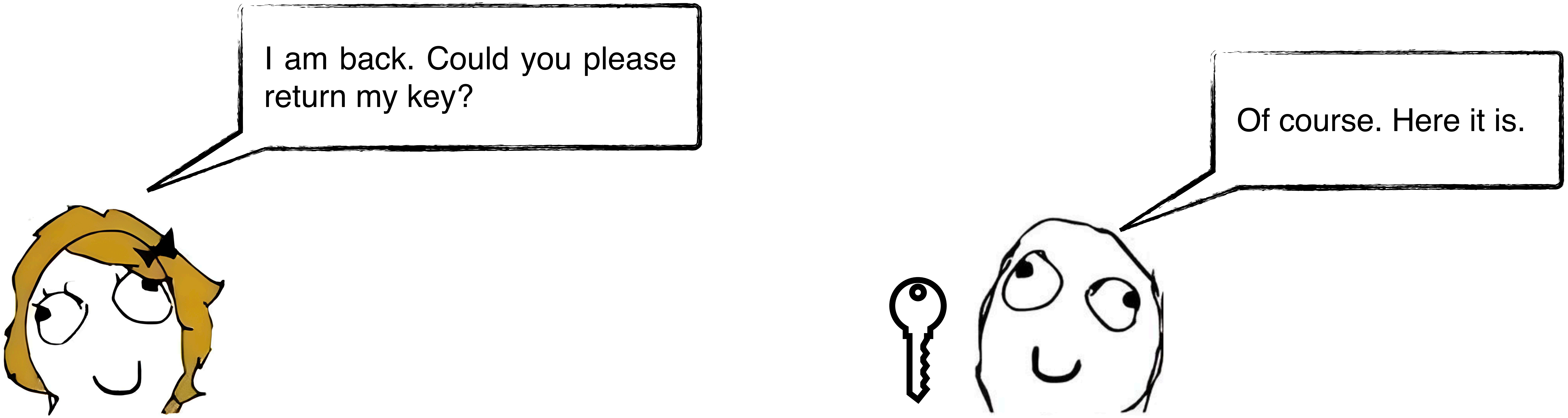

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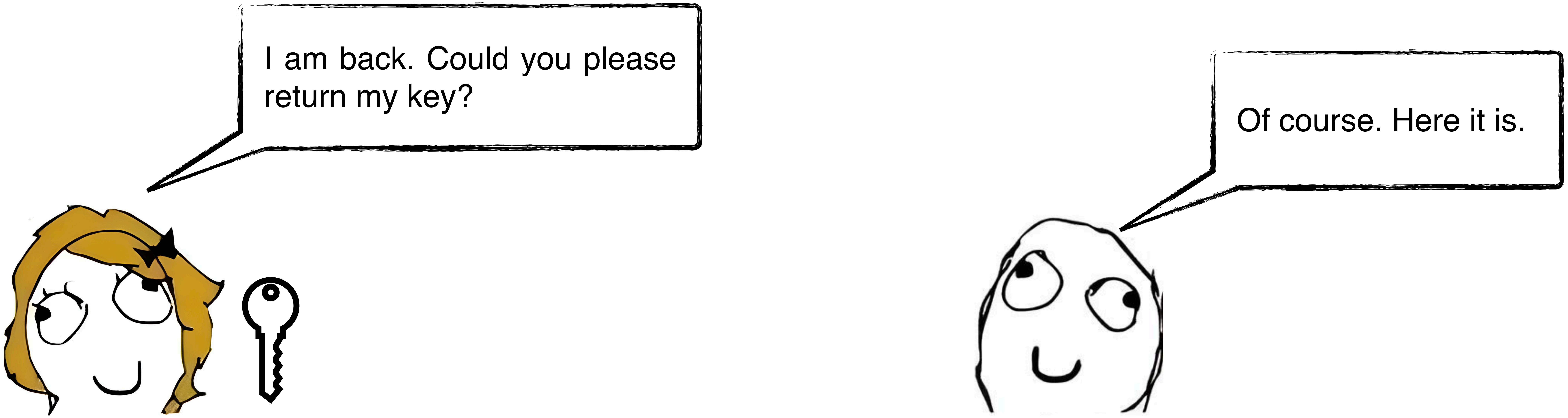

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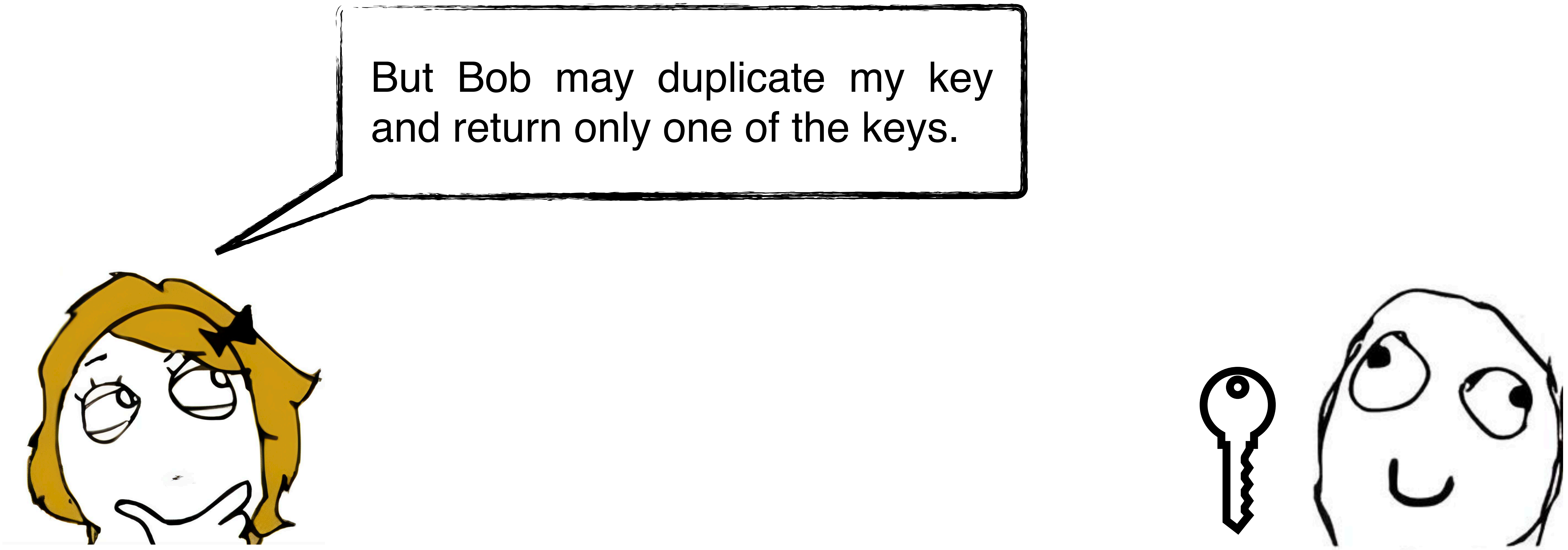

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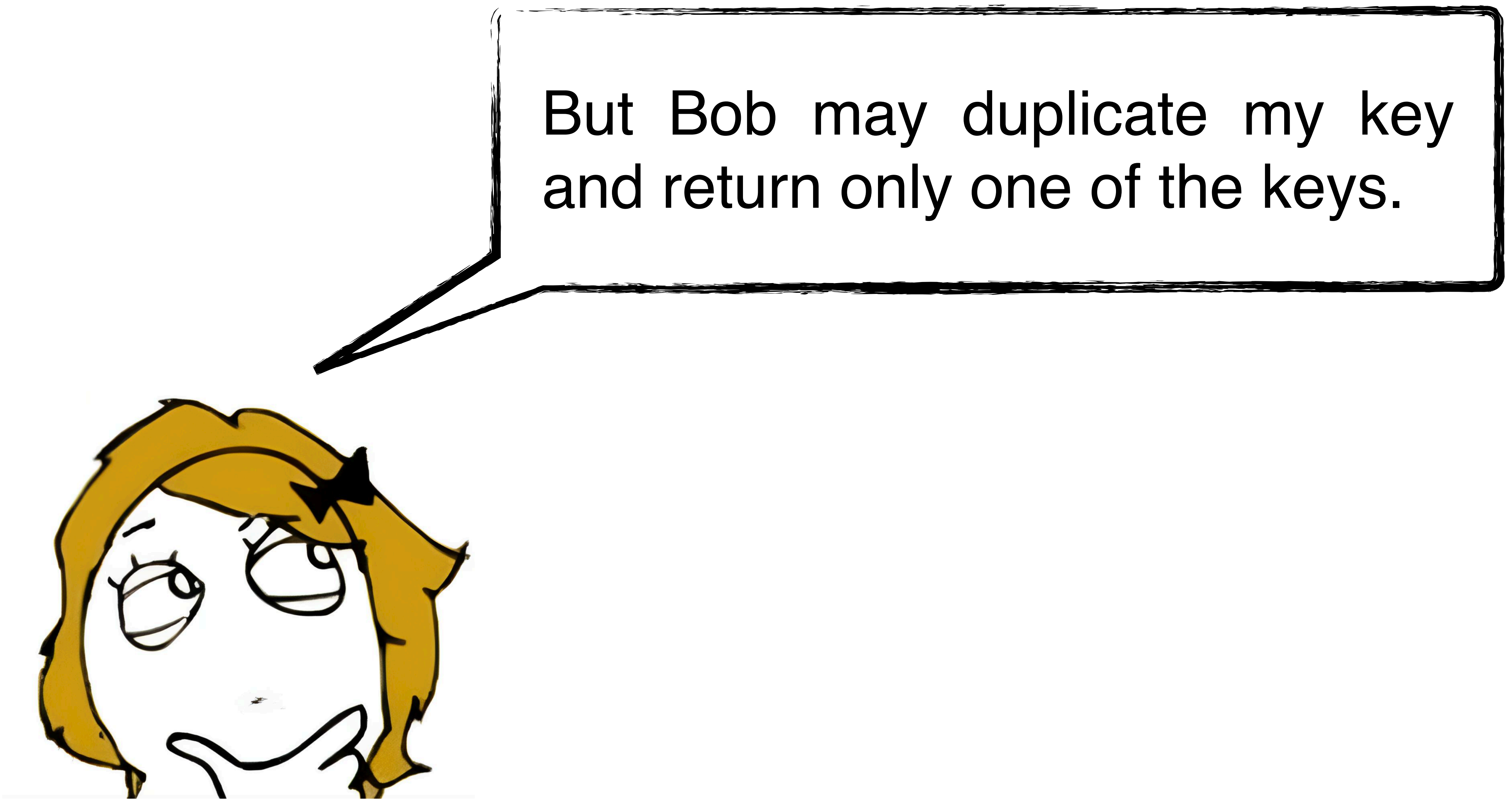

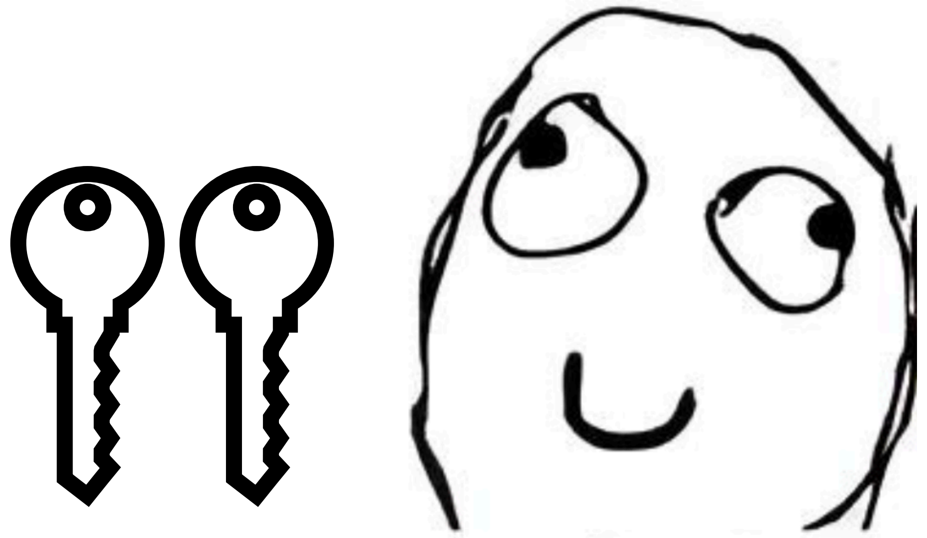

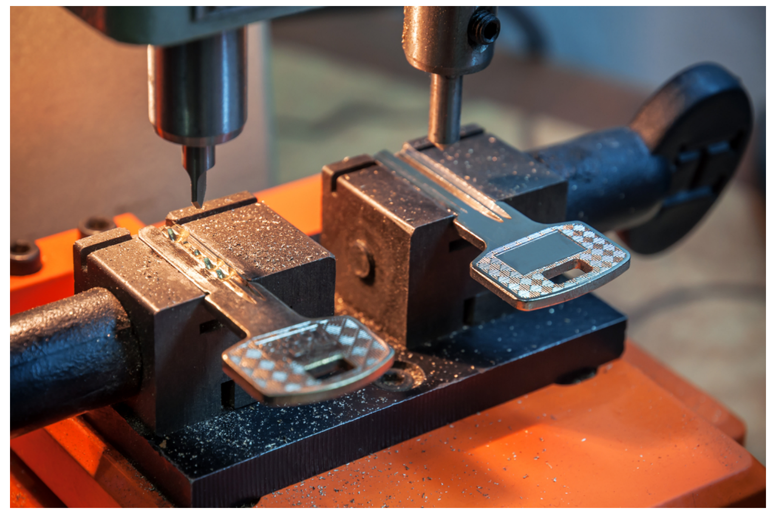

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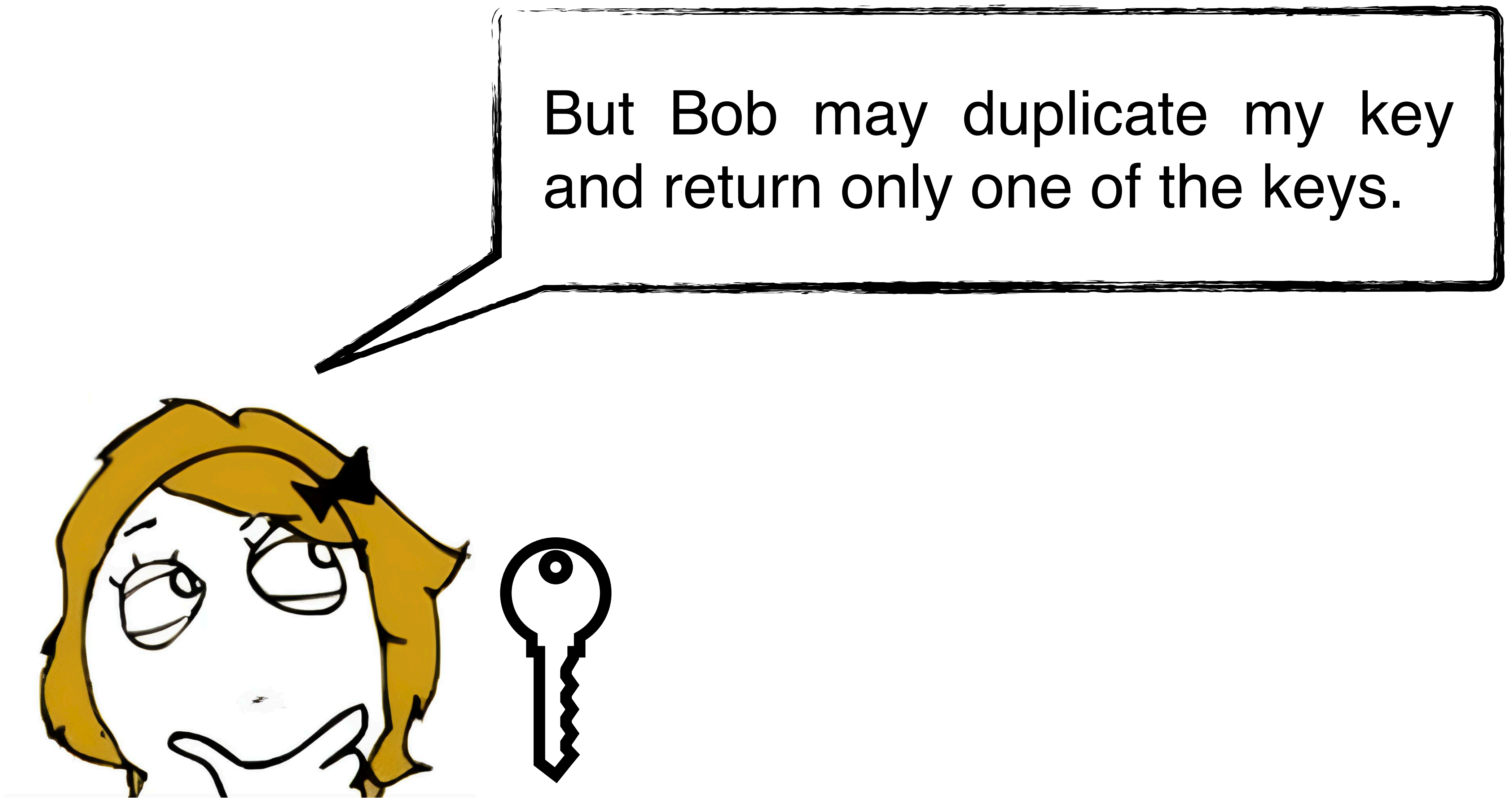
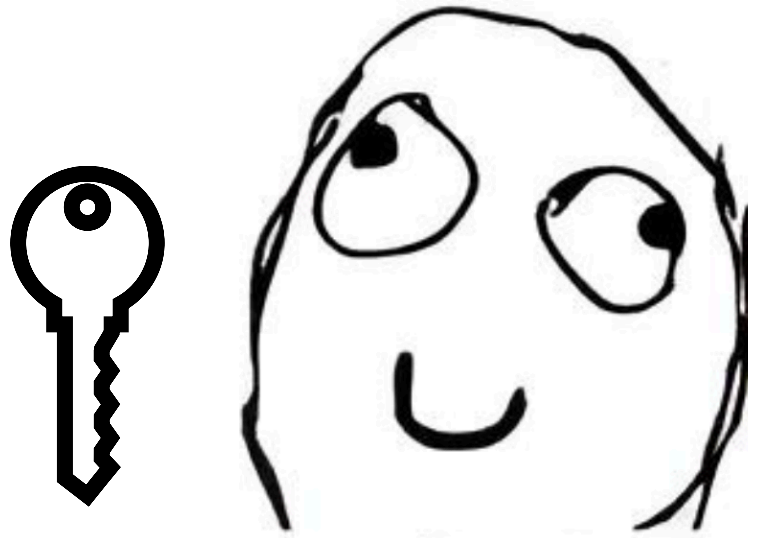
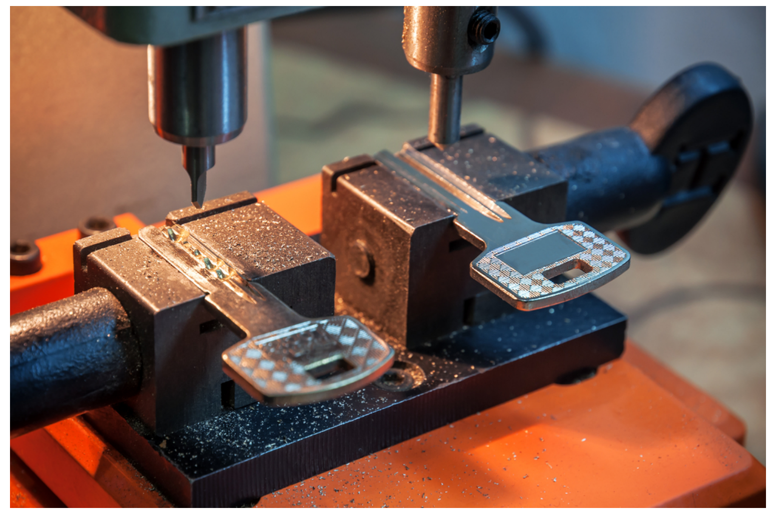


Bob may have access to Alice's mailbox after returning one of the keys.





What is revocable cryptography?

Leverage the no-cloning principle of quantum mechanics to delegate and revoke cryptographic capabilities enabled by secret keys.

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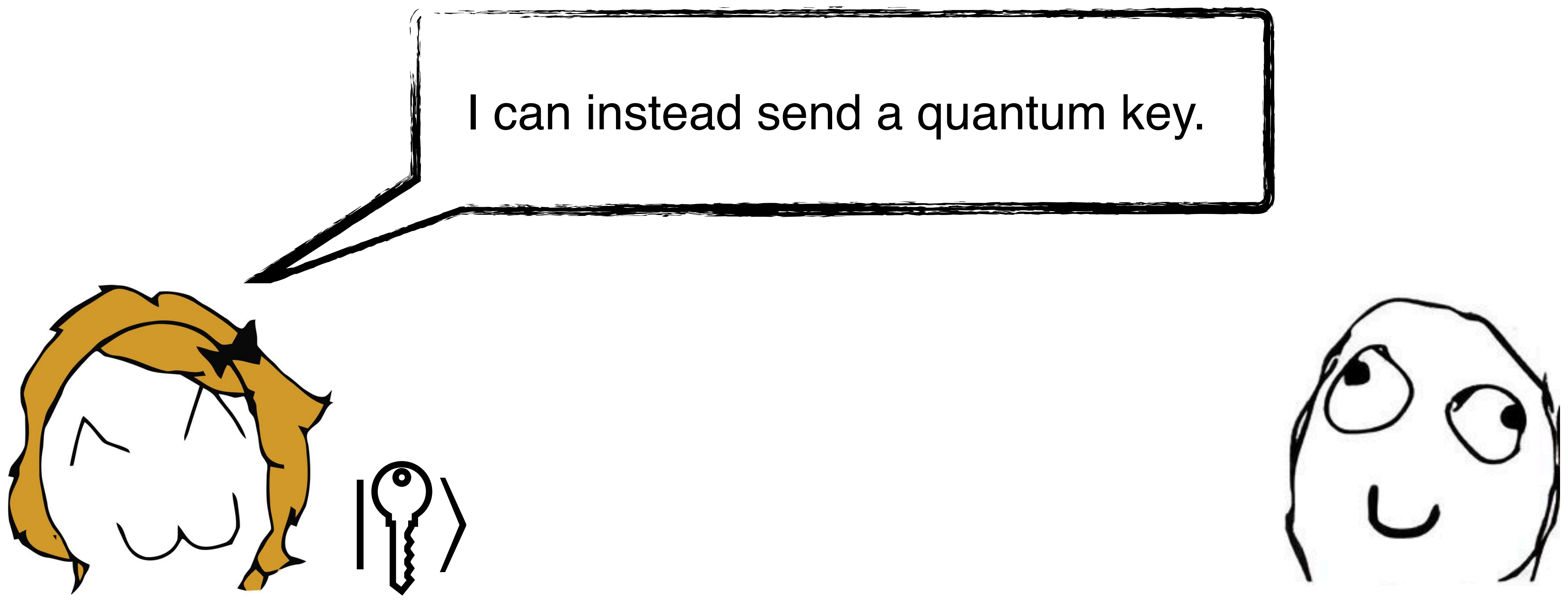
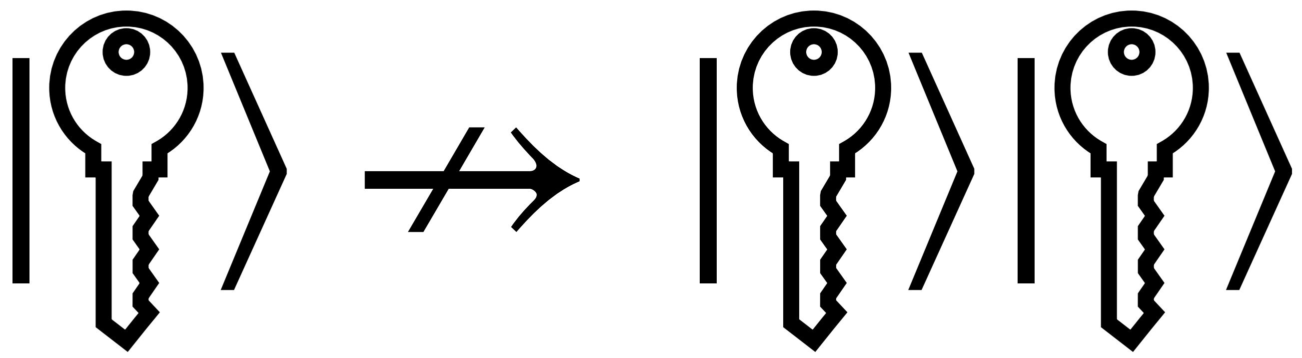


No-Cloning Theorem:



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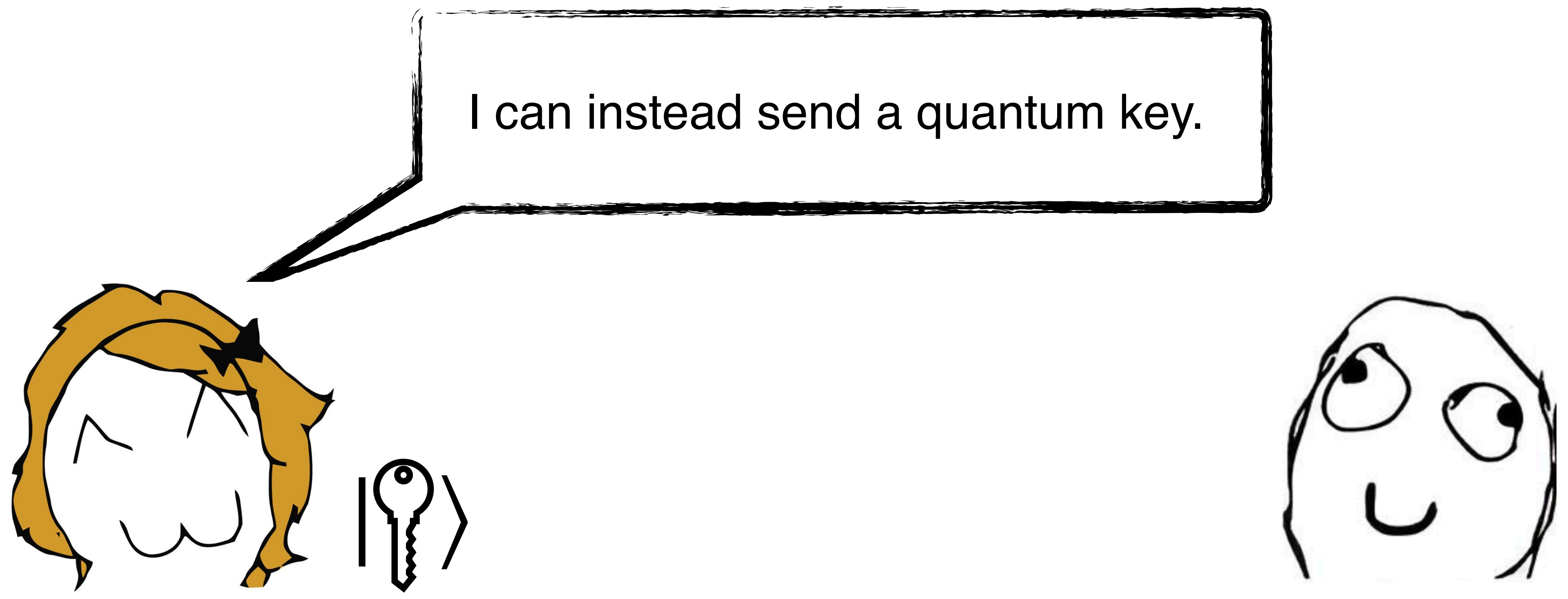

- It’s weaker than copy-protection [Aar09], yet meaningful, and can be based on weaker assumptions
- Unlike cryptography with certified deletion [BI20, HMNY21, BK22], an honest user is supposed to return the original quantum key for revocation

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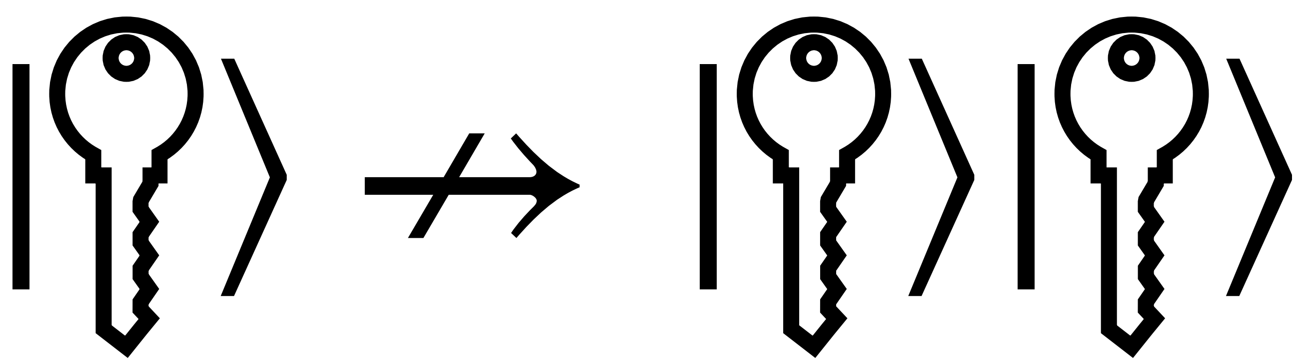

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Correctness:

(1) with $|$$\rangle$, Bob has the cryptographic capabilities

(2) honest Bob can pass the check Revoke

Security:

After sending a state that passes the check Revoke, Bob no longer has the cryptographic capabilities

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Revocable FHE

Revocable PRF

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The ability to decrypt

Revocable public-key encryption

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The ability to decrypt

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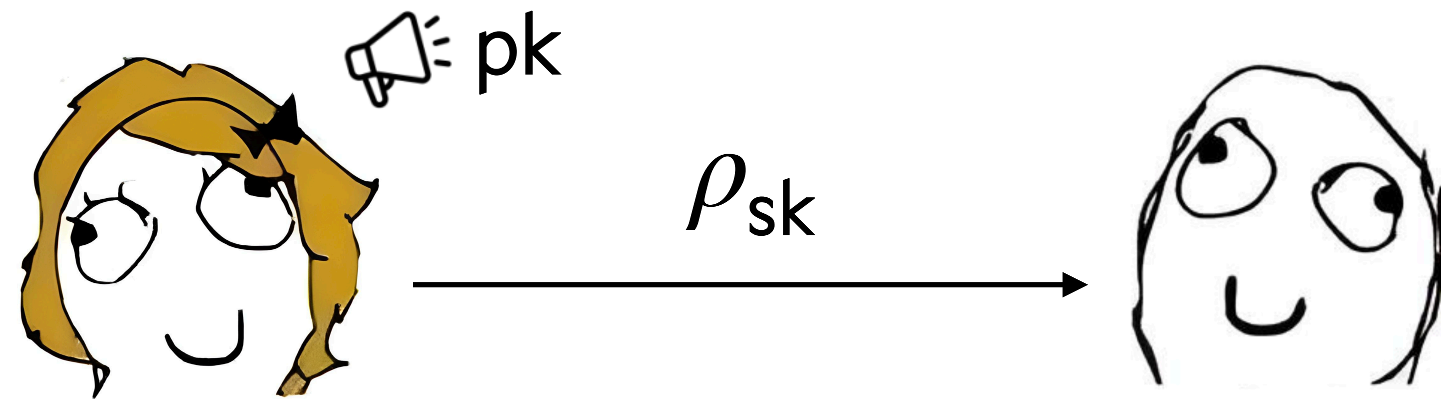
The ability to evaluate

...

Revocable PKE: Syntax

Leverage quantum mechanics to delegate and revoke the ability to decrypt.



$\mathsf{KeyGen}(1^\lambda) \to (\mathsf{pk}, \mathsf{msk}, \rho_{\mathsf{sk}})$

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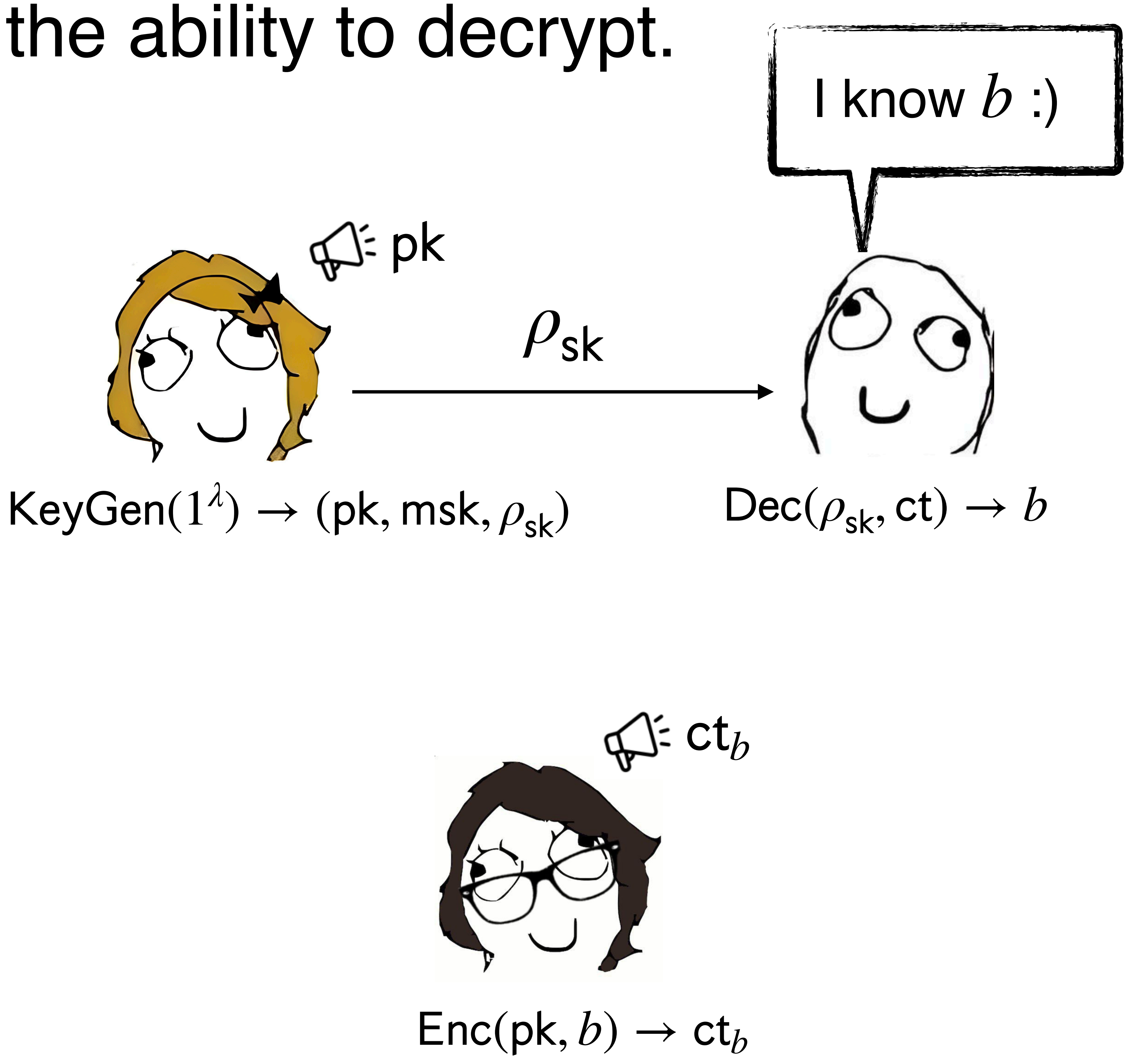

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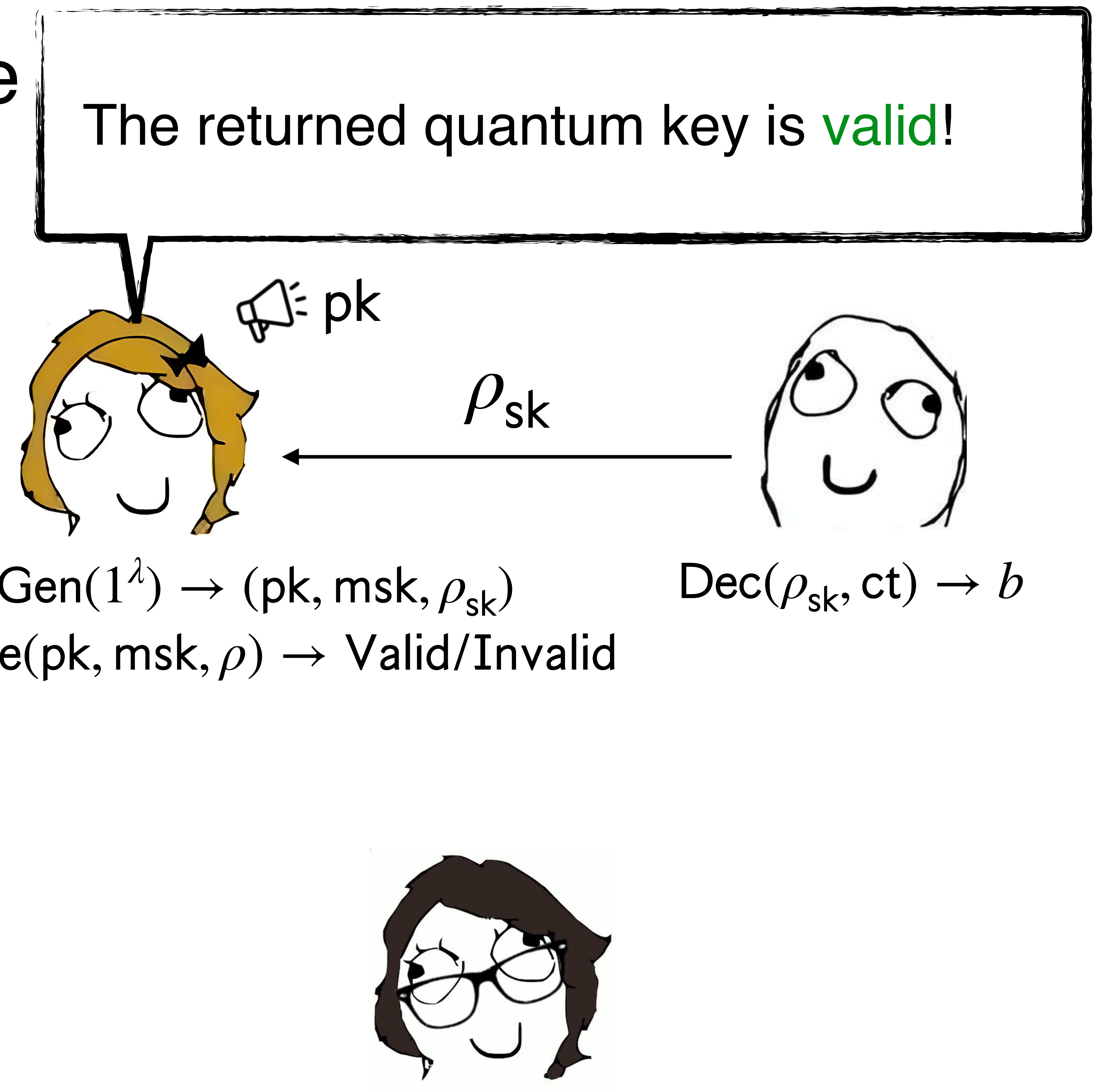

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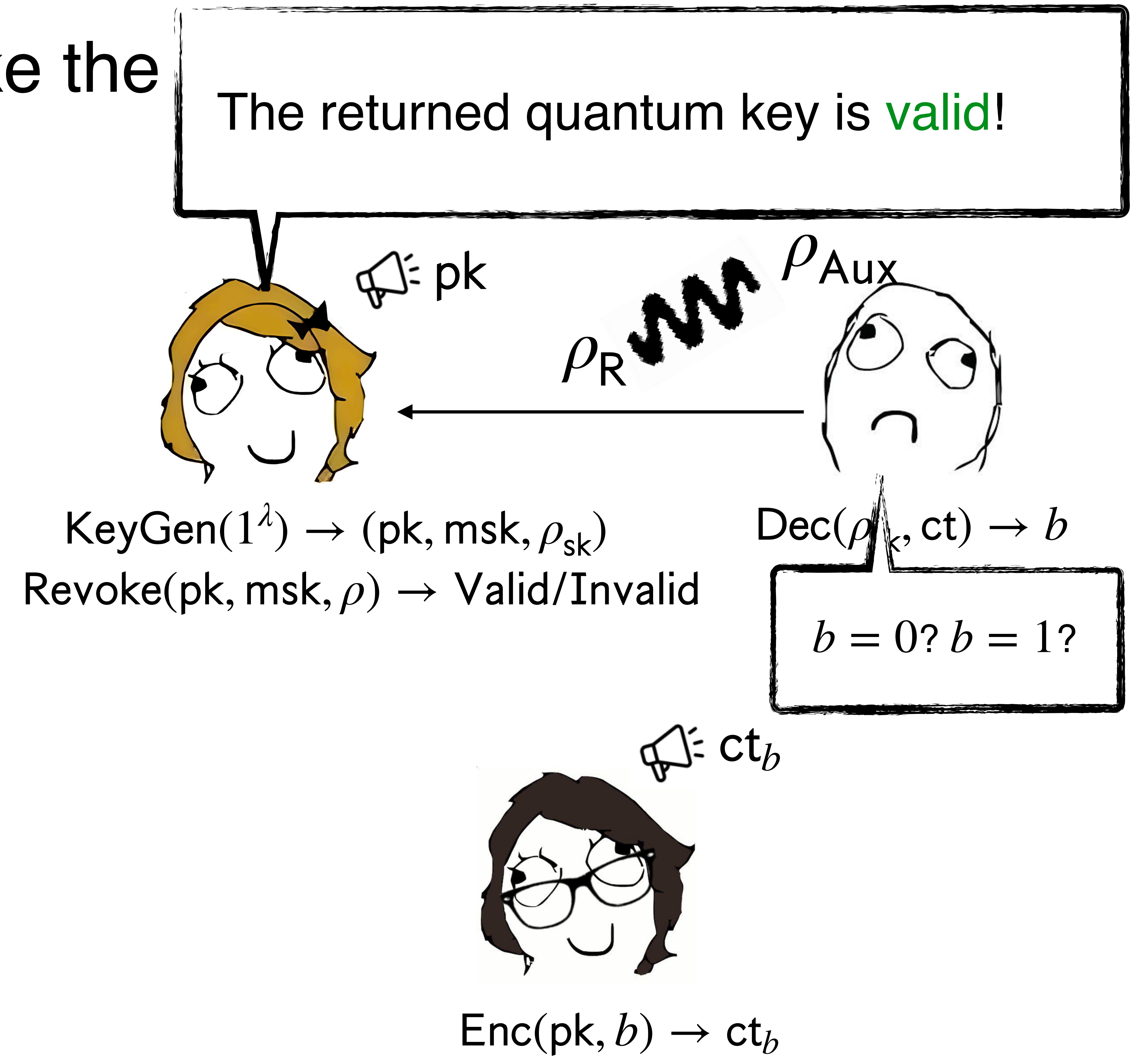

Correctness:

(1) with |  ⟩, Bob can decrypt

(2) honest Bob can pass the check Revoke

Security:

After sending a state that passes the check Revoke, polynomial-time Bob can no longer distinguish encryption of 0 and encryption of 1

It suffices to consider encryption for a bit $b \in \{0,1\}$.

Prior work

- Assuming post-quantum PKE, there exists a revocable PKE scheme [AKN+23]
- Assuming simultaneous dual-Regev conjecture, the dual-Regev PKE scheme is revocable [APV23]
- Assuming post-quantum sub-exponential hardness of LWE, there exists a revocable PKE scheme with classical revocation [CGJL23]

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[APV23]: Can we prove that the dual-Regev PKE scheme is revocable from LWE?

Why do we care about the dual-Regev PKE scheme?

(1) [APV23] gave many reductions from revocable dual-Regev PKE scheme!

(2) It's a textbook PKE, and may inspire other protocols with similar structures.

Our work

Assuming post-quantum polynomial hardness of LWE over sub-exponential modulus,

- The dual-Regev PKE scheme (the construction in [APV23]) is revocable

+ the results in [APV23]

Assuming post-quantum polynomial hardness of LWE over sub-exponential modulus,

- The dual-Regev PKE scheme has classical revocation
- There exists revocable FHE with quantum/classical revocation
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The first revocable PRF from concrete assumptions

Recall: Dual-Regev PKE

The public key is $(\mathbf{A}, \mathbf{y})$ for a random matrix $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$ and some $\mathbf{y} \in \mathbb{Z}_q^n$

To encrypt: $\mathsf{Enc}(\mathsf{pk}, b) = (\mathbf{s}^T\mathbf{A} + \mathbf{e}^T, \mathbf{s}^T\mathbf{y} + b\left\lfloor\frac{q}{2}\right\rfloor + e')$

<u>The classical decryption key</u>:

A short preimage $\mathbf{x}$ such that

$$\mathbf{A}\mathbf{x} = \mathbf{y}$$

To decrypt: Notice that $\mathbf{s}^T\mathbf{y} + b\left\lfloor\frac{q}{2}\right\rfloor + e' - \left(\mathbf{s}^T\mathbf{A} + \mathbf{e}^T\right)\mathbf{x} \approx b\left\lfloor\frac{q}{2}\right\rfloor$

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A superposition of short preimages $\mathbf{x}$ such that

$$|\varphi_{\mathbf{y}}\rangle = \sum_{\mathbf{x} \in \mathbb{Z}_q^m, \mathbf{A}\mathbf{x} = \mathbf{y}} \rho_\sigma(\mathbf{x}) \, | \, \mathbf{x}\rangle$$

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To revoke: use msk (a short basis of $\mathbf{A}$) to check whether the returned state is $|\varphi_{\mathbf{y}}\rangle$

The route in [APV23]

The public key is $(\mathbf{A}, \mathbf{y})$ for a random matrix $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$ and some $\mathbf{y} \in \mathbb{Z}_q^n$

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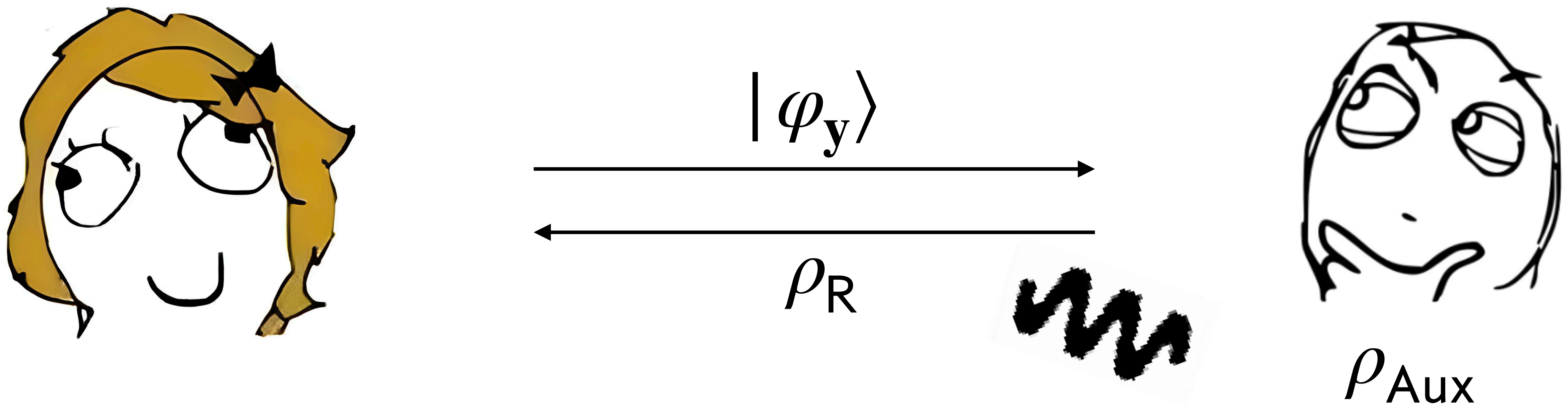

$(\mathbf{s}^T\mathbf{A} + \mathbf{e}^T, \mathbf{s}^T\mathbf{y} + e')$ vs $(\mathbf{u}, r)$

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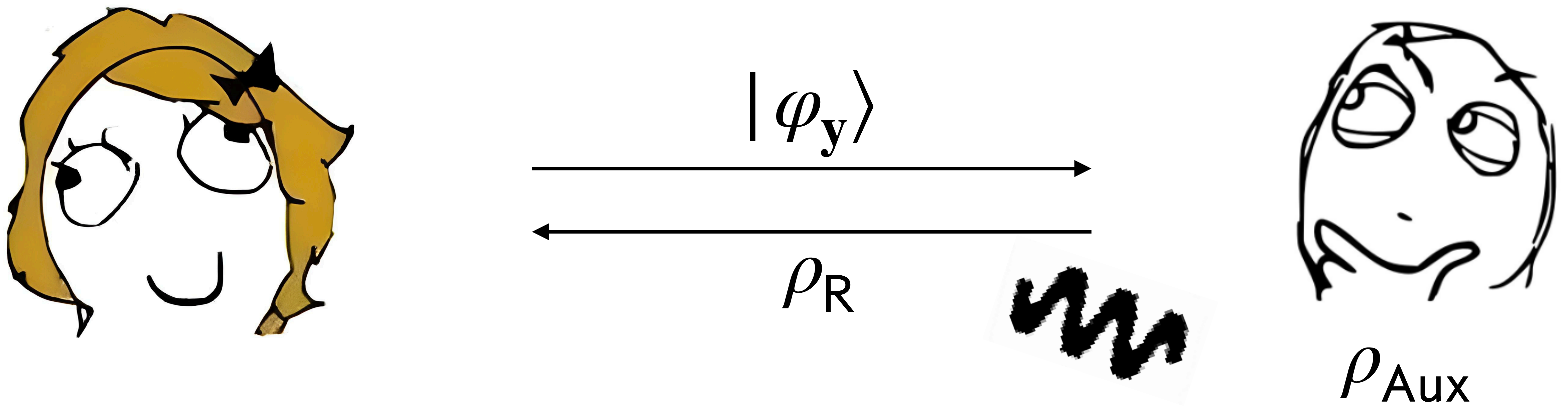

$(\mathbf{s}^T\mathbf{A} + \mathbf{e}^T, \mathbf{s}^T\mathbf{y} + e')$ vs $(\mathbf{u}, r)$

Extract a short preimage $\mathbf{x}_0$ from R and a short preimage $\mathbf{x}_1$ from Aux

Then use $\mathbf{x}_0 - \mathbf{x}_1$ to break SIS!

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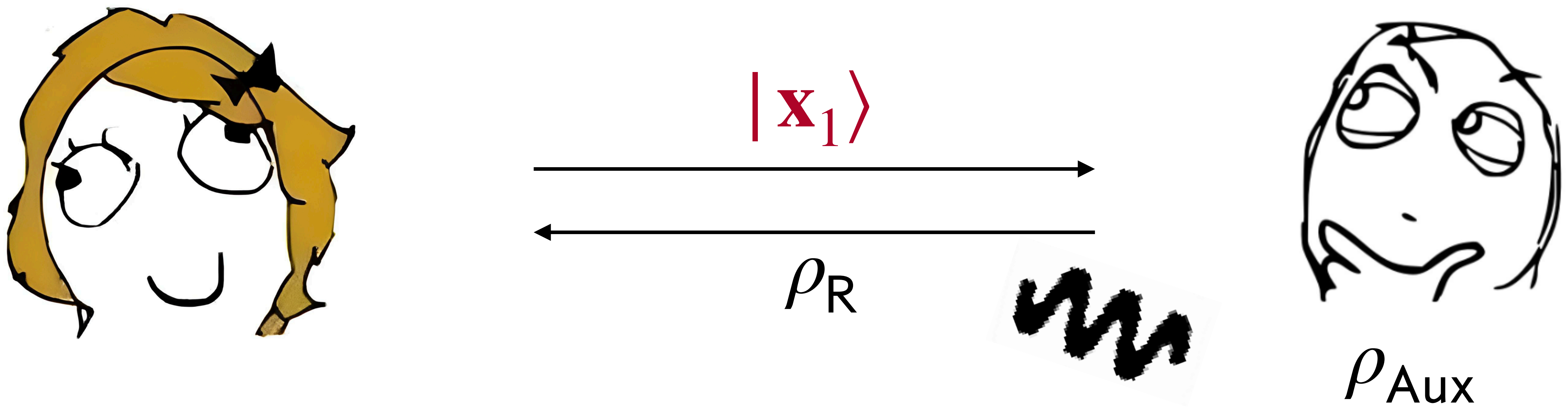

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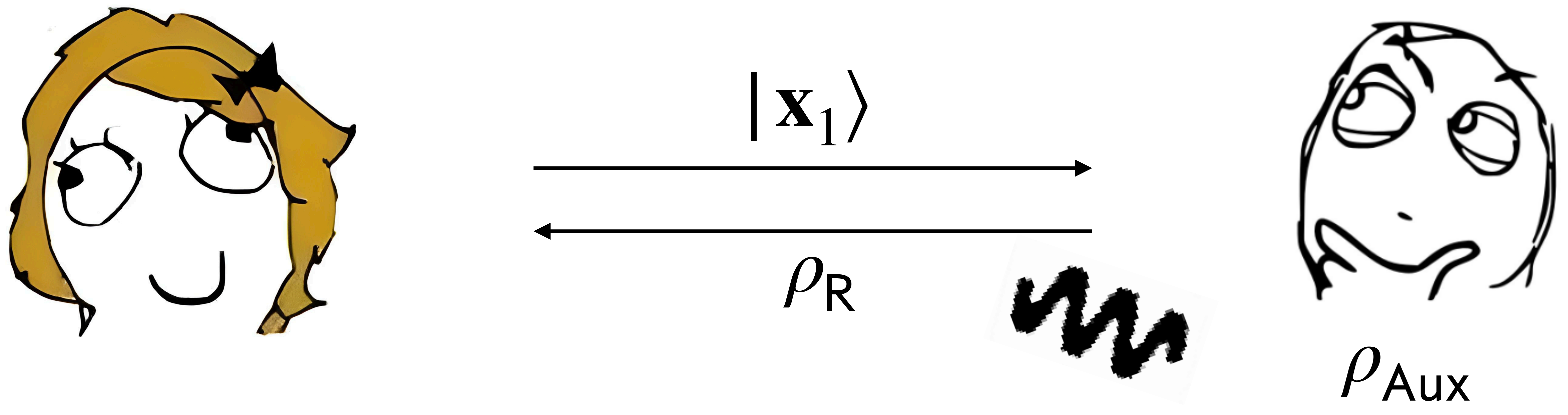

$(\mathbf{u}, \mathbf{u}^T\mathbf{x}_1 + e')$ vs $(\mathbf{u}, r)$

Extract a short preimage $\mathbf{x}_1$

$$\mathbf{Ax}_1 = \mathbf{y}$$

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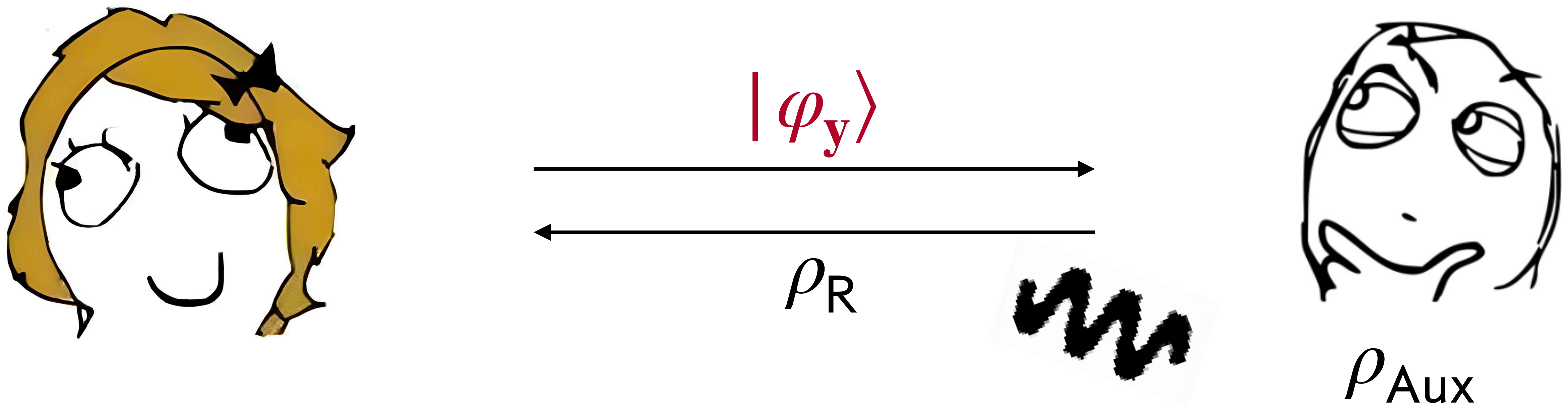

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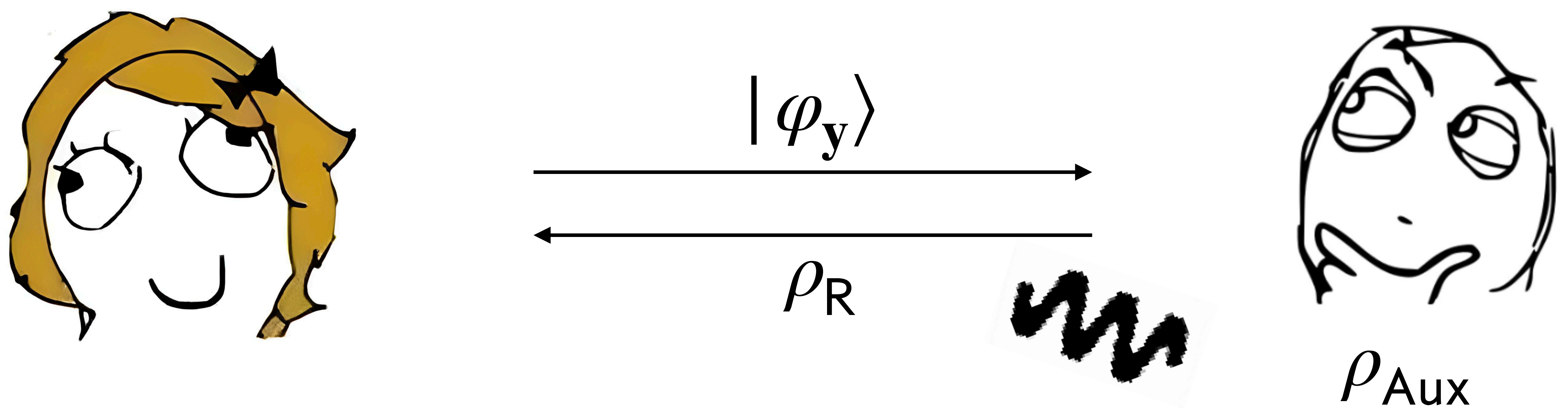

Revoke passes: $\rho_{\mathrm{R}} \approx |\varphi_{\mathbf{y}}\rangle\langle\varphi_{\mathbf{y}}|$

$(\mathbf{u}, \mathbf{u}^T\mathbf{x}_1 + e')$ vs $(\mathbf{u}, r)$

Computational Measurement => a short preimage $\mathbf{x}_0$

$$\mathbf{A}\mathbf{x}_0 = \mathbf{y}$$

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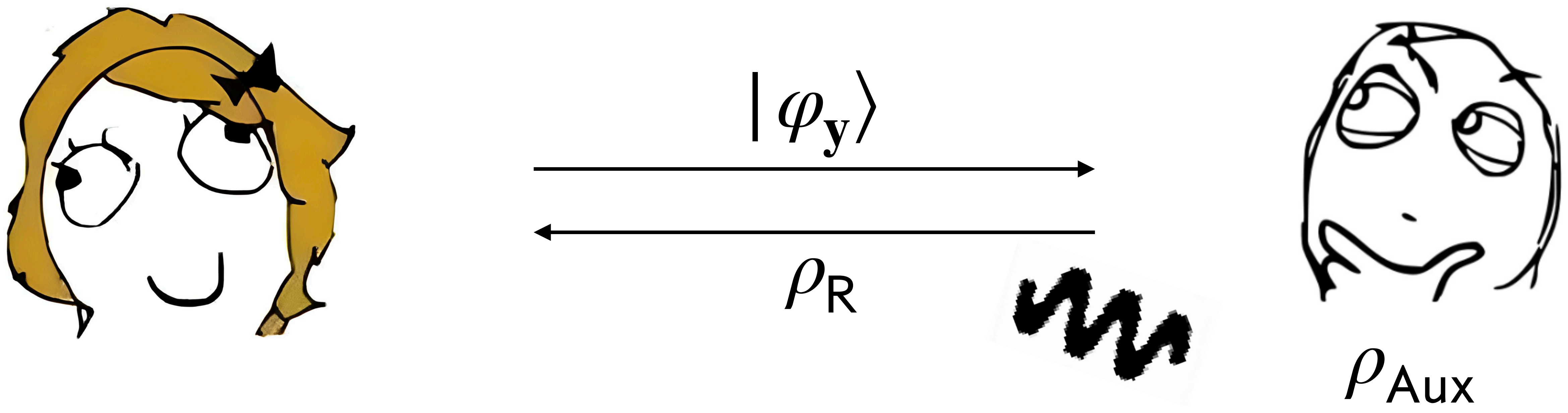
Challenge: get $\mathbf{x}_0$ and $\mathbf{x}_1$ simultaneously?



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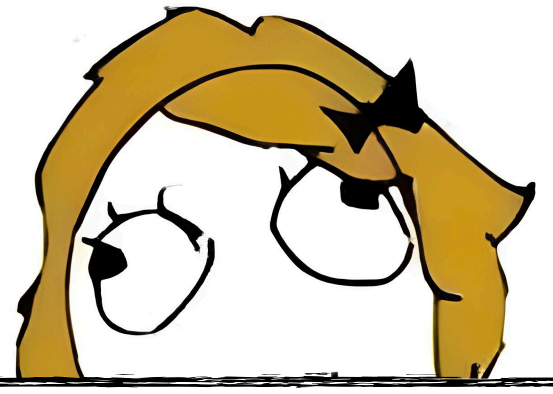
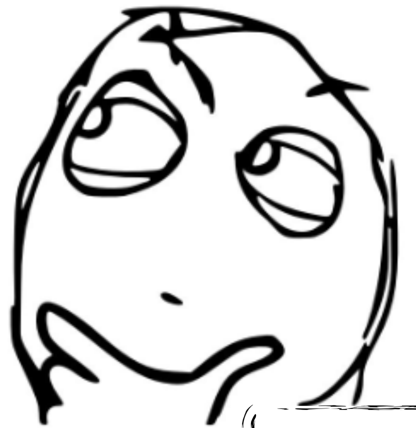


$|\varphi_{\mathbf{y}}\rangle$ →

← ρ_{R}



ρ_{Aux}

Revocation succeeds w.p. $1/\text{poly}(n)$

Extraction succeeds w.p. $1/\text{poly}(q)$

Revoke passes: $\rho_{\text{R}} \approx |\varphi_{\mathbf{y}}\rangle\langle\varphi_{\mathbf{y}}|$

$(\mathbf{u}, \mathbf{u}^T\mathbf{x}_1 + e')$ vs $(\mathbf{u}, r)$

Computational Measurement => a short preimage $\mathbf{x}_0$

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Extract a short preimage $\mathbf{x}_1$

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Our approach: almost perfect extraction

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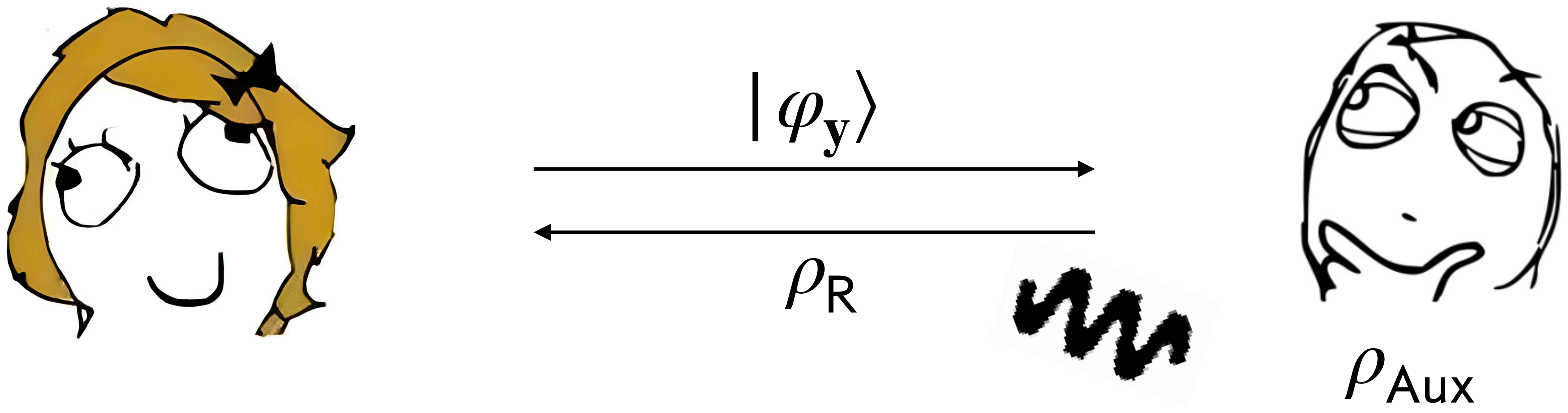

- Test if it is a good distinguisher for $(\mathbf{s}^T\mathbf{A} + \mathbf{e}^T, \mathbf{s}^T\mathbf{y} + e')$ vs $(\mathbf{u}, r)$ via ATI

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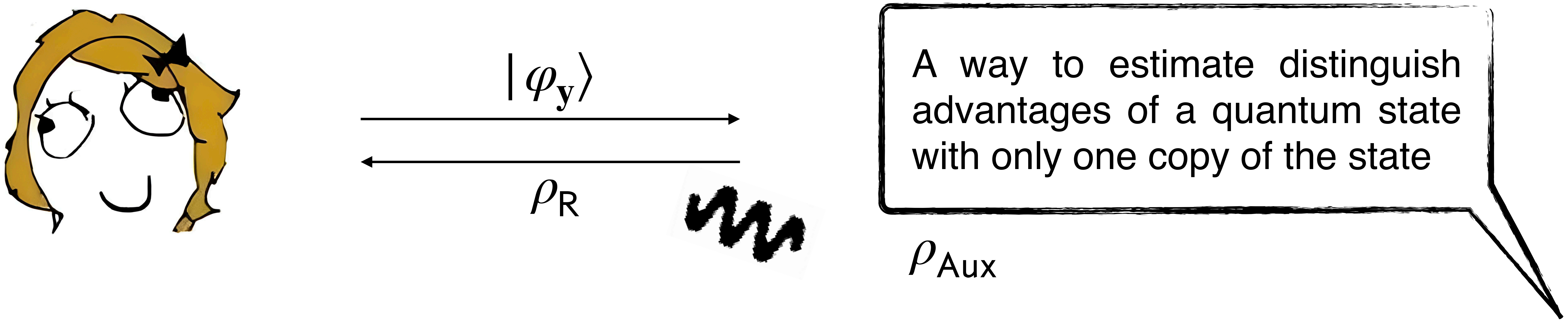

- Test if it is a good distinguisher for $(\mathbf{s}^T\mathbf{A} + \mathbf{e}^T, \mathbf{s}^T\mathbf{y} + e')$ vs $(\mathbf{u}, r)$ via ATI

Our approach: almost perfect extraction

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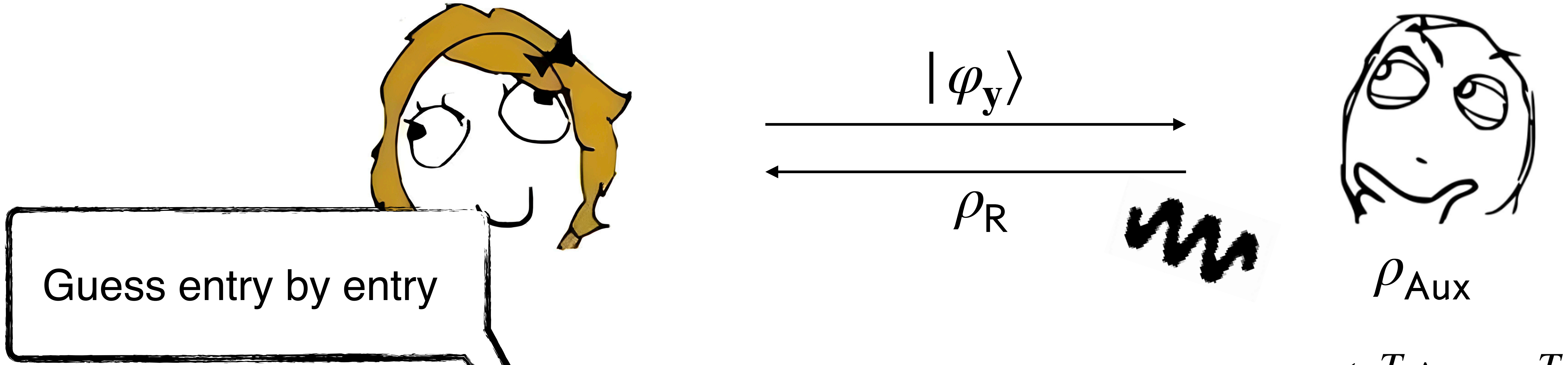

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- If yes, test it on $(\mathbf{s}^T\mathbf{A} + \mathbf{e}^T + c\mathbf{i}, \mathbf{s}^T\mathbf{y} + e' + c \cdot g)$ vs $(\mathbf{u}, r)$ for each guess g
  - If the i^{th} entry of $\mathbf{x}_1$ is g, it's a good distinguisher with certainty
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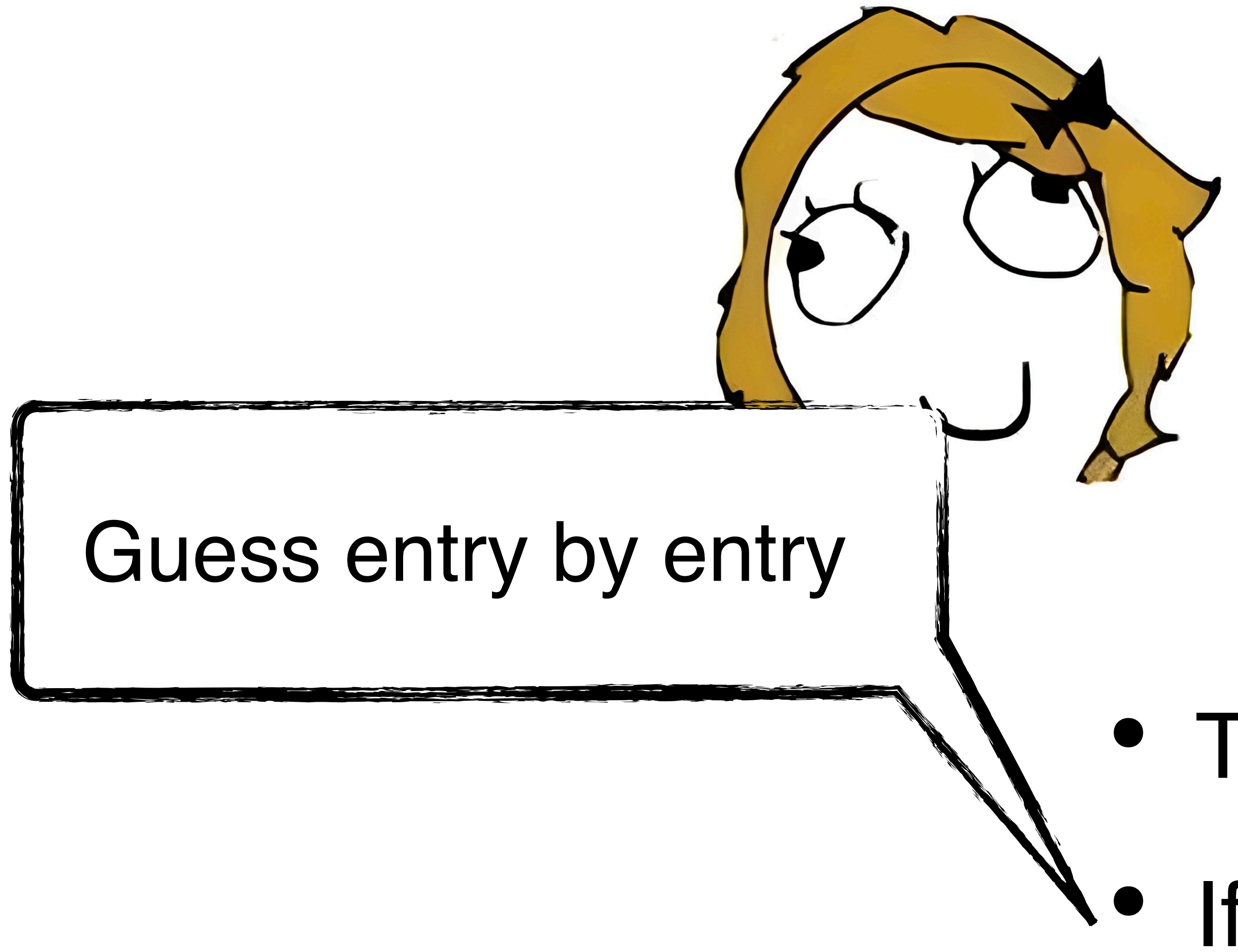

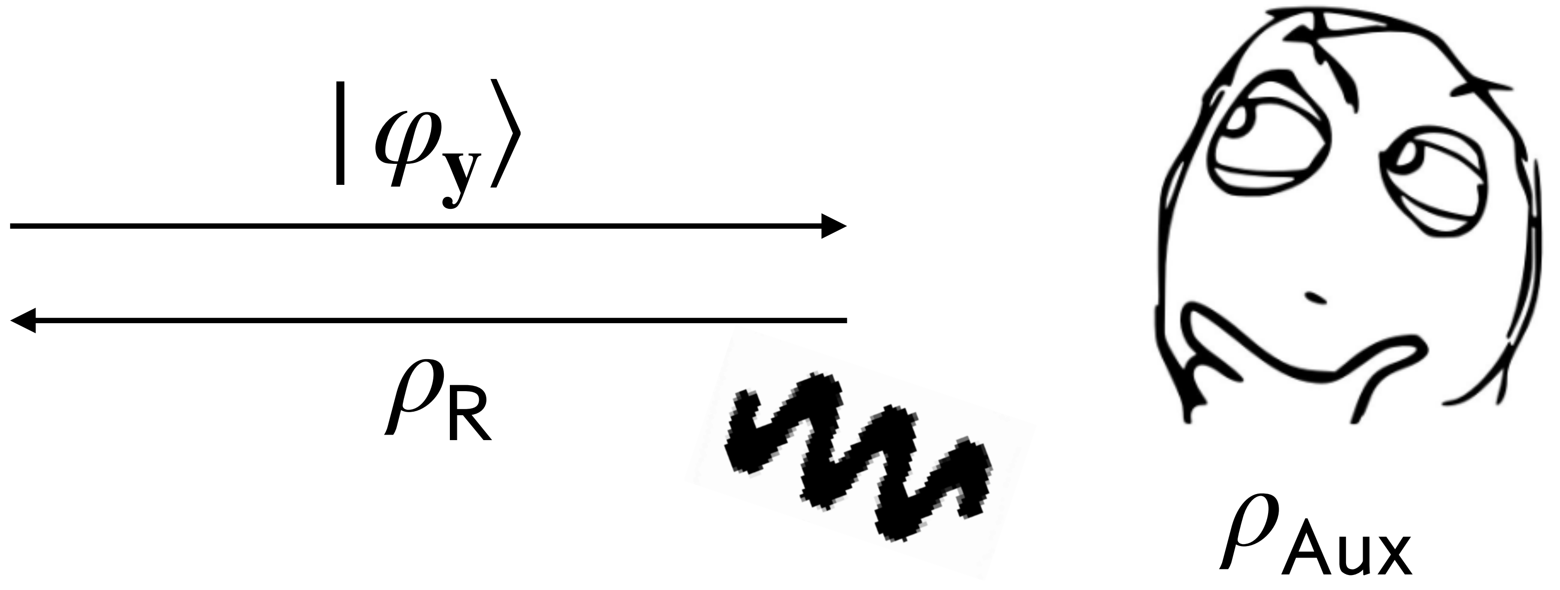

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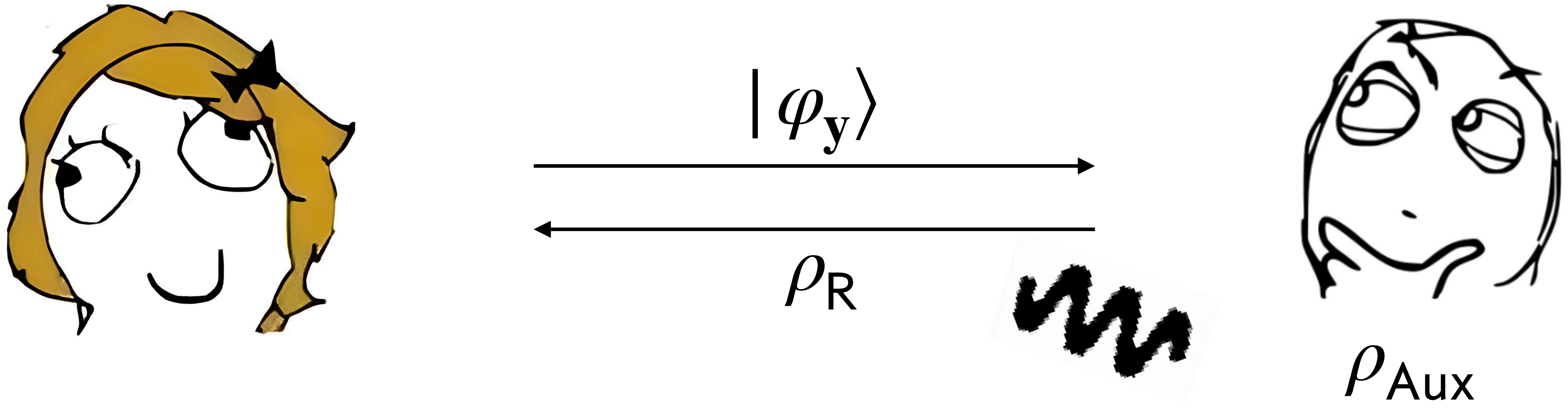

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As long as the first test passes, the extraction works with certainty.

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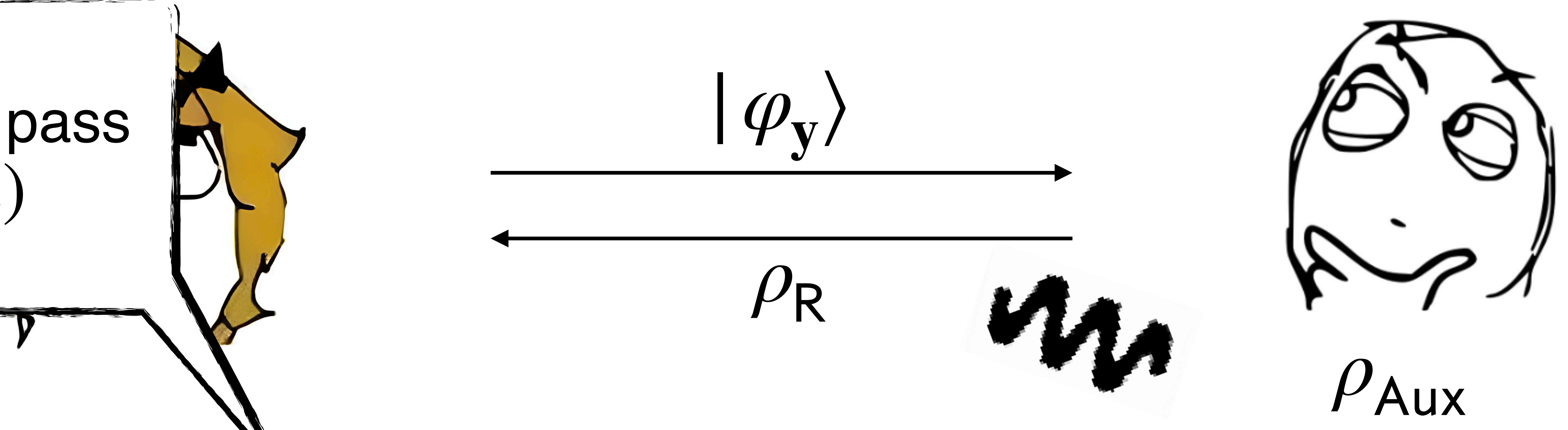
We can get $\mathbf{x}_0$ and $\mathbf{x}_1$ simultaneously!

The test and the revocation pass simultaneously w.p. $1/\text{poly}(n)$



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Conclusion

Assuming post-quantum polynomial hardness of LWE over sub-exponential modulus,

- The dual-Regev PKE scheme (the construction in [APV23]) is revocable

+ the results in [APV23]

Assuming post-quantum polynomial hardness of LWE over sub-exponential modulus,

- The dual-Regev PKE scheme has classical revocation
- There exists revocable FHE with quantum/classical revocation
- There exists revocable PRF with quantum/classical revocation

Thank you!

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